3. Force and Gravity

Being in orbit is like being infatuated – you are constantly falling, but you aren't getting closer.

(LEO errors corrected 31 January 2006)
(Black hole discussion expanded 2 Feb 2006)
(Equation for Gravity added 2 Feb 2006)
(Error in 100 km gravity corrected 2 Feb 2006)

Gravity surprises

When orbiting the Earth, the head of an astronaut is "weightless." When he sneezes, it will snap back . . .

exactly as much as it does when he is sitting on the Earth's surface

At an altitude of 100 km, you are virtually at the edge of space, since more than 99.999% of the Earth's atmosphere is below you. At this altitude, the force of gravity is less than at the Earth's surface. It is smaller by . . .

about 3%.

The $10 million "X Prize" was awarded last year to the first private company to send a rocket to an altitude of 100 km. Some think this is the beginning of private exploitation of space. But getting to 100 km takes less energy than it takes to get into orbit around the Earth. How much less?

about 30 times less.

If the Sun suddenly turned into a black hole, but its mass didn't change (it is presently about 300,000 times the mass of the Earth), then the orbit of the Earth . . .

wouldn't change.

Those facts surprise most people. That's because they misunderstand several important concepts, including weightlessness, orbits, and the behavior of gravity.

The Force of Gravity

Any two objects that have mass attract each other with a force we call gravity. You probably never noticed this for small objects because the force is so weak. But the Earth has lots of mass, and so it exerts a big gravitational force on you. We call that force your *weight*. The fact that gravity is actually a force of attraction is not obvious. Prior to the work of Isaac Newton, it was assumed that gravity was simply the natural tendency of objects to move downward.

If you weigh 150 lb, and are sitting about 1 m (3.3 ft) from another person of similar weight, then the gravitational force of attraction between the two of you is 10^{-7} lb. This seems small, but such forces can be measured; it is about the same as the weight of a flea.

You weigh less when you stand on the Moon, because the Moon doesn't put as big a force on you. If you weigh 150 lb on the Earth, you would weigh only 25 lb on the Moon. You haven't changed (you are made up of the same atoms), but the force exerted on you is different. Physicists like to say that your *mass* hasn't changed, only your weight. Think of mass as the amount of material, and weight as the force of attraction of gravity.

Mass is commonly measured in kilograms. If you put a kilogram of material on the surface of the Earth, the pull of gravity will be a force of 2.2 lb. So a good definition of a kilogram is an amount of material that weighs 2.2 lb when placed on the surface of the Earth. That number is worth remembering, since kilograms are commonly used around the world.[1]

Suppose you weigh 150 lb on the Earth. Then your mass is about 68 kg (i.e. it is 150/2.2). Go to the surface of Jupiter, and you will weigh nearly 400 lb. On the surface of the Sun you will weigh about 2 tons (T), at least for the brief moment before you are fried to a crisp. But in all cases your mass will be 68 kg.

The equation that describes the pull of gravity between two objects was discovered by Isaac Newton and is called Newton's law of gravity. It says that the force of attraction is proportional to the mass--double the mass and the force doubles. The force also depends on a special relationship with the distance called an *inverse square* law. The law is inverse because when the distance gets larger, the force gets smaller. The law is square because if you triple the distance, the force decreases by nine; if you make the distance increase by 4, then the force goes down by 16, etc.

This law is usually written in the following way:

$$F = GMm/r^2.$$

M is the mass of the pulling object, m is the mass of the object being pulled, and r is the distance between them. G is a constant that makes the units come out right. You don't have to memorize this. I put it here because I will use it several times in this chapter to make some calculations, but I don't require you to be able to duplicate these calculations.

For large objects like the Earth, some of the mass may be very close (right under your feet) and some can be far away (on the other side of the Earth). It turns out that to get the right answer for a spherical object, you can just use the distance to the center and you'll

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[1] A more accurate value is that there are 2.205 lb in 1 kg, and 0.4536 kg in 1 lb, but don't bother memorizing these more precise numbers.

get the right answer. If you are standing on the surface of the Earth, then the right value of r to use will be the Earth's radius, about 4000 miles (6000 km). If you are in a satellite, then the distance to the center is the height (distance above the surface) plus the radius. I will not ask you to make such calculations, but I explain this in case you want to understand some of the calculations I'll do in this chapter, such as those for a black hole.

As you get further away from the surface, r increases, and the gravity decreases. If you are 4000 miles above the surface of the Earth, so your distance to the center has doubled, then the force decreases by a factor of 4.

If you go up only 100 km, your distance to the center has changed by less than 2%. Gravity is weaker, but only by 3%. For details, I do an optional calculation in the footnote.[2]

Newton's Third Law

Here is something that might surprise you: if you weigh 150 lb, not only is the Earth attracting you with a force of 150 lb, but you are attracting the Earth with a force of 150 lb too. This is an example of Newton's third law, which states: if an object exerts a force on you, then you exert the same force back on it. In my mind, this law is so fundamental that it should have been Newton's first law. (Newton's first law is that an object in motion tends to stay in motion unless there is an outside force. Newton's second law, F = ma, is one we'll get to soon.)

But if you are so small, how can you exert such a large force on the Earth? The answer is that, even though you are small, your mass exerts a force on every piece of the Earth, simultaneously. When you add all those forces together, the sum is 150 lb. So you are pulling up on the Earth exactly as much as the Earth is pulling down on you.

Think of it in the following way: if you push on some else's hand, they feel your force. But you feel the force too. You push on them; they push back on you. The same thing works with gravity. The Earth pulls on you; you pull on the Earth.

The "Weightless Astronaut" Paradox

Imagine an astronaut in orbit in a capsule 200 km (125 mi) above the surface of the Earth. If the astronaut weighed 150 lb on the Earth, what does he weigh now? Because he is further away from the mass of the Earth, the force is slightly lower. From Newton's law of gravity, we can calculate that the astronaut weighs 142 lb, i.e. he is 8 lb lighter.

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[2] The radius of the Earth is 6371 km. (We use the approximate value of 6000 in the body of this text.) If you are at an altitude of 100 km, then your distance to the center has changed to 6471. The value of $1/r^2$ changes from 2.46E-8 to 2.39E-8. That is a change of 3%. Everything else in the force equation is unchanged, so your total force also changes by 3%.

But wait a minute--aren't orbiting astronauts weightless? Movies show them floating around inside spaceships. How can they do that if they weigh almost as much as they do when they are on the surface of the Earth?

To understand the answer to this paradox, we have to think about what it means to be weightless. Suppose you are in an elevator, and the cable suddenly breaks. The elevator and you fall together. During those few seconds before you crash into the ground, you will feel weightless. You will float around inside the elevator. You will feel no force on your feet, and your shoulders will not feel the weight of your head. (Your head falls with your chest at the same rate, so the muscles in your neck aren't needed to keep your head above your chest.) In those brief seconds you have the same "weightless" experience as the astronauts. All the time, of course, the Earth is pulling you rapidly towards it.[3] You have weight, but you feel weightless. A movie made of you in the elevator would show you floating around, apparently without weight, while you and the elevator fell together. You would look just like the astronauts floating around in the International Space Station.

Now imagine that instead of falling, the elevator is shot out of a gun with you inside and it flies 100 mi before hitting the ground. During that trip, you will again feel weightless. That's because you are in motion along with the elevator. You and it fly in the same arc.[4] Your head and chest are both moving in that arc together; there is no force between them and your neck muscles can be completely relaxed. Your head will seem to have no weight. Prior to the sending them into orbit, potential astronauts were flown in airplanes following such arcs in order to see how they responded to the sensation of weightlessness and to get them used to it.

When you and the elevator are moving together under the force of gravity (either falling or shot in an arc) there seems to be no gravity. From that alone, you might think you were far out in space, far away from the gravity of any planet, star, or moon. From inside the elevator, you can't tell the difference.

Now imagine that at the top of a very tall tower (200 km high) is a large gun, pointing horizontally. Out of this gun, we are going to shoot the elevator, with you inside. If we pick a low velocity (e.g. 2 km/sec) you and the elevator will curve towards the Earth, and crash into it, as in path *A* in the figure below. But if instead we pick a higher velocity like 8 km/sec, you and the elevator will follow path *B*; you will curve towards the Earth, but because of your high velocity, you will miss the edge of the Earth. You will keep on curving downward, but you will never hit. You are in orbit. The force of gravity makes the path of the elevator--let's call it a space capsule now--curve downwards. But if that

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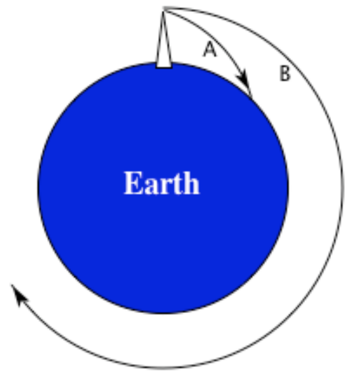
[3] There are rides at some amusement parks that allow you to fall for long distances and experience weightlessness, at least for a small number of seconds. We'll calculate how many seconds in a later section.

[4] Your path could be traced by a geometric curve known as a parabola.

curvature matches the curvature of the Earth, then it misses the surface, and stays at a constant height.[5]



A capsule shot into space from a tower

This may seem preposterous, but it is reasonable to think of an astronaut in orbit around the Earth as being in a state of perpetual falling. That's why he feels weightless.

You can think of the Moon as doing the same thing. It is attracted to the Earth by gravity, but it has high sideways motion. Even though it is falling towards the Earth, it always misses.

The velocity for LEO (Low-Earth Orbit)

To stay in a circular orbit just a few hundred miles above the surface, the velocity of a satellite must be about 8 km/s, which is about 18,000 mi/h. (The actual value depends slightly on the altitude; we'll derive this number from a calculation later in this chapter.) At this velocity, the satellite orbits the 24,000-mile circumference of the Earth in about 1.5 h.

Remember these numbers: **a satellite orbiting in a LEO (low-Earth orbit) goes 8 km/s and takes an hour and a half to go around the Earth.**

If the astronaut inside a satellite wants to land, he does *not* fire his rockets in a direction away from the Earth; he fires his rockets towards the direction he is headed--in the forward direction! The force of the rocket exhaust slows down the satellite, so it is no longer going fast enough to miss the edge of the Earth. If he fires the rockets enough to stop the satellite completely, then the satellite will simply fall straight downward. Gravity brings the satellite back to Earth. If the satellite moves faster than 8 km/s, it will leave the circular orbit and head out into space. At about 11 km/s it will have sufficient

[5] If your velocity is not exactly horizontal, or if your velocity is a little low or high, then the orbit will not be a circle but an ellipse.

velocity to reach the Moon and beyond. This velocity is called the escape velocity. We'll discuss this concept further later in this chapter.

Analogy with a Rock and Sling

There is another way to think about Earth satellites. Forget gravity for a moment. Imagine that you have a rock tied at the end of a string, and you are spinning it in a circle above your head. The string provides the force that keeps the rock from flying away, and this force also keeps the rock in circular motion. If the string breaks, the rock flies off in a straight line. Gravity does the same thing for an Earth satellite: it provides the force that keeps the satellite in a circular orbit.

An old weapon called the "sling" is based on this principle. A rock is held by a leather strap and spun in circles over the head. Arm motion helps it pick up circular speed. It is the strap that keeps the rock in circular motion. When the strap is released, the rock flies in a straight line towards its target. Such a sling was the weapon that, according to the Bible, David used to kill the giant Goliath.

In a similar manner, if we could suddenly "turn off" the force of gravity, the Moon would leave its circular orbit, and head off in a straight line. The same is true for all the satellites now in orbit around the Earth. And with the Sun's gravity turned off, the Earth would head out into space too, at its previous orbital speed of 30 km/s (67,500 mi/h).

Geosynchronous Satellites

Weather satellites and TV satellites have a very special orbit: they are *geosynchronous*. This means that they stay above the same location of the Earth at all times. It also means that the same weather satellite will be able to watch the development of a storm, or of a heat wave, continuously. And, if you are receiving a signal for your TV, you never have to re-point the antenna because the satellite remains in the same direction above your house at all times.

How can this be, since satellites must orbit the Earth to avoid crashing back down? The answer is elegant: the satellites orbit the Earth at such a high altitude (where the gravity is weak) that they go at a low velocity, and take 24 h for each orbit. Since the Earth rotates once in that period, they stay above the same location. Both are moving--your home with the TV dish, and the satellite--but their angle with respect to each other doesn't change.

Geosynchronous satellites orbit the Earth at the very high altitude of 22,000 mi. That distance is over 5 times the radius of the Earth. The force of gravity is an "inverse square" law. That means that if the distance is 5 times larger, then that factor makes the force 25 times smaller. Moreover, at a high altitude, the distance to make a circular orbit is longer. These factors combine to make the time to circle the Earth equal to 24 h, rather than the 1.5 h of LEO satellites.

There is a catch. If the satellite is to stay in the same location in our sky relative to the ground, it must orbit above the equator. Can you see why that is true? A geosynchronous satellite moves in a circle around the center of the Earth. If the satellite is not in an equatorial orbit, then it will spend half of its orbit in the Northern Hemisphere and half in the Southern. Only if it orbits above the equator can it stay precisely above the same Earth location at all times.

As a result, all geosynchronous satellites are right above the equator. If you look at them up in the sky, they all line up in a narrow arc. In fact, there is so little room left, that international treaties are required to divide up the space. (If satellites are too close to each other, their radio signals can interfere.)

If you are kidnapped, and don't know where you have been taken, try to spot a satellite dish. If the dish is pointing straight up, then you know you are on the Equator. If it is pointing horizontally, then you know you are at the North Pole.[6]

Spy Satellites

Spy satellites carry telescopes to look down on the surface of the Earth and see what is going on. They were once used exclusively by the military, to see the secrets of adversaries, but now they are widely used by government and industry to look at everything from flooding and fires to the health of food crops.

The ideal spy satellite would stay above the same location all the time. But to do that, it must be geosynchronous, and that means that its altitude is 22000 mi. At those distances, even the best telescopes can't see things smaller than about 200 m. (We'll derive that number when we discuss light.) That means that such a spy satellite could see a football stadium, but couldn't tell if a game were being played. Such satellites are good enough to watch hurricanes and other weather phenomena, but are not useful for fine details, such as finding a particular terrorist.

Thus, to be useful, spy satellites must be much closer to the Earth. That means they must be in LEO (low-Earth orbit), no more than a few hundred miles above the surface. But if they are in LEO, then they are not geosynchronous. In LEO they zip around the 24,000-mile circumference of the Earth in 1.5 h; that gives them a velocity relative to the surface of 16,000 mi/h. At this velocity, they will be above a particular location (within ± 100 mi of it) for only about a minute.[7] This is a very short time to spy. In fact, many countries that want to hide secret operations from the United States keep track of the positions of our spy satellites, and make sure their operations are covered over or hidden during the brief times that the spy satellite is close enough to take a photo.

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[6] But be careful. Even at the equator, the satellite doesn't have to be overhead. The satellite could be above the Congo and you could be in Brazil. So you really have to determine the direction of north to make good use of the satellite dish information.

[7] At 16,000 mi/h = 4.4 miles/sec, it will go 200 mi in 45 seconds.

LEO satellites cannot hover. If they lose their velocity, they fall to Earth. If you want to have continuous coverage of a particular location, you must use a circling airplane, balloon, or something else that can stay close to one location.

The military and intelligence agencies are developing quiet, high-flying drones (airplanes without a pilot) to do the most critical kind of spying. But these vehicles can be shot down more easily than a satellite.

GPS--A Medium-Earth Orbit (MEO) Satellite

One of the wonders of the last decade is the Global Positioning System (GPS). A small GPS receiver costs under $100 and it will tell you your exact position on the Earth within a few meters. I've used such a receiver in the wilderness of Yosemite, in the souks of Fez, and in the deserts of Nevada. You can buy a car with a built-in GPS receiver that will automatically display a map on your dashboard showing precisely where you are. The military uses GPS to steer its smart bombs to land at just the location they want, within a few meters.

A GPS receiver picks up signals from several of the 24 GPS satellites currently orbiting the Earth. It is able to determine the distance to each satellite by measuring the time it took for a signal to go from the satellite to the receiver. Once the receiver has measured the distance to three satellites, it can then calculate precisely where on Earth it is located.

To understand how the GPS receiver calculates its position, consider the following puzzle: A person is in a U.S. city. He is 800 mi from New York City, 900 mi from New Orleans, and 2200 mi from San Francisco. What city is he in?

Look on a map. There is only one city that has those distances, and that is Chicago. Knowing three distances uniquely locates the position. GPS works in a similar manner, but instead of measuring distances to cities, it measures distances to satellites. And even though the satellites are moving, their locations when they broadcast their signals are known. They tell the computer in your GPS receiver where they are, so the GPS receiver can figure out where it is.

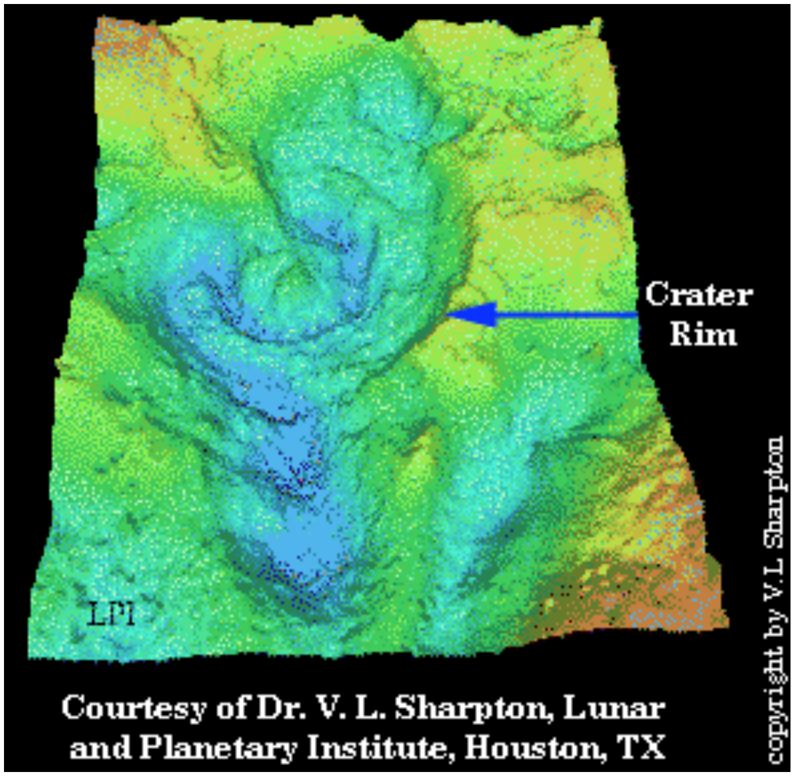
You might expect that the GPS satellites would be in geosynchronous orbits. But they are not, primarily because such a large distance would require that their radio transmitters have much more power to reach the Earth. They were not put in low orbit (LEO) because they would often be hidden from your receiver by the horizon. (Each one would be in view for, typically, 7 min *if* it passed directly overhead.) So they were placed in a medium-Earth orbit (MEO) about 12,000 mi high. They orbit the Earth every 12 h.

Using gravity to search for oil

I said earlier that every object exerts a small gravitational force on every other object. Remarkably, measurement of such small forces has important practical applications. If you are standing over an oil field, you feel slightly less gravity than the gravity you feel standing over solid rock. Oil is less dense than rock; it has less mass (per cubic kilometer), and so its gravity isn't as strong.[8] Such small gravity changes can even be measured from airplanes flying above the ground. An instrument can make a "gravity map" that shows the density of the material under the ground. Maps of the strength of gravity, taken by flying airplanes, are commonly used by businesses to search for oil and other natural resources.

A more surprising use of such gravity measurements was to make a map of the buried Chicxulub crater on the Yucatan peninsula, the crater left behind when an asteroid killed the dinosaurs. The crater was filled in by sedimentary rock that was lighter than the original rock, so even though it is filled, it shows a gravity "anomaly," i.e. a difference from what you would get if the rock were uniform. An airplane flying back and forth over this region made sensitive measurements of the strength of gravity, and they are represented in the map shown below. In this map, the tall regions are regions in which the gravity is stronger than average, and the low regions are locations where the gravity is slightly weaker.

[8] Optional footnote: The gravity of a spherical object acts as if it all originates from the center of the object. But this is true only if the sphere's mass is uniformly distributed. If we treat the Earth as such a sphere, then you can mathematically think of the mass of an oil field as a sum of a uniform Earth, and a little bit of "negative" mass that cancels out some of the gravity. If you are close to the oil field, you will sense the reduced gravity because this little bit of negative mass will not attract you as much as if it were denser rock.



The crater shows several concentric circles, with the largest over 100 km in diameter. The inner rings probably formed when material from under the huge crater was forced upward, partially filling it.

Manufacturing objects in a weightless environment

When the space program began, many people thought that there would be significant advantages to being in a weightless environment. In a satellite, things wouldn't sag under their own weight. It might be possible to make better (rounder) ball bearings, or to grow more perfect crystals (used in computers and other electronics) in a weightless environment.

This promise has been largely unfulfilled. The additional cost of doing the work in a satellite has not turned out to be worth it. It costs about $10,000 to launch a kilogram of anything into orbit. In the near future, commercial companies hope to reduce that price to $1,000/kg. It's hard to make a profitable factory in space when it costs that much just to get there. There is no reason in principle why getting to space *must* be expensive; we'll show in a later section that the energy required is only 15 Cal/g. Some time in the future, if travel to space becomes as cheap as an airplane ride, then the idea of factories in orbit may be more likely.

Gravity on the Moon and Asteroids

Our Moon has about 1/81 of the mass of the Earth. So you might think its gravity would be 81 times less. But the Moon's radius is 3.7 times smaller than that of the Earth. Remember that gravity follows an inverse square law, so from the small radius you would expect the force to be $(3.7)^2 = 13.7$ times larger. If you combine these two effects, you get surface gravity 13.7/81 = 1/6 that on the Earth. And that is the value the astronauts found when they landed there. The gravity is so weak that they seemed to bound around in slow motion. When they jumped, they went high, and when they came back down, the came down slowly.

What is the surface gravity of an asteroid that has a radius of 1 km? We can't say, since we don't know the mass. However, if you assume that the density is the same as for the Earth, then we can derive a simple result: the surface gravity is proportional to the radius.[9] Since the radius of the Earth is 6378 km, this means that the surface gravity on the asteroid is 1/6378 of the value on the Earth. If you weigh 150 lb on Earth, you would weigh 150/6378 = 0.023 lb (about 1/3 oz) on the asteroid. That's about the weight of 3 pennies on the Earth.

As we'll show in a later section on escape velocity, you would have to be very careful on the asteroid. Your escape velocity would be very low and you could easily launch yourself into space by jumping. To jump into space from the Earth, you would require a velocity of 11 km/s; from the asteroid, you would only require a velocity of 2 m/s. (The same jump speed would get you less than a foot high on the surface of the Earth.) This low escape velocity was a problem for a U.S. space probe called the Near Earth Asteroid Rendezvous (NEAR). If the satellite landed at a velocity of 2 m/s or more, then it might have bounced right back out into space.

Gravity in science fiction

One of the most common "errors" in science fiction movies is the implicit assumption that all planets in all solar systems have a gravity about equal to the Earth's gravity. There is no reason why that should be so. Pick a random planet, and you are just as likely to be a factor of 6 times lighter (and bouncing around like astronauts on the Moon) or 6 times heavier and unable to move because of your limited strength. Imagine a person who weighs 150 lb on the Earth trying to move on a planet where he weighs 900 lb.

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[9] Optional footnote: That's because the mass of the asteroid M = density *x* volume. Call the density d. The volume of a sphere is $(4/3)\,\pi\, R^3$. So the gravity at the surface is $F = GMm/R^2 = G\,m\,(4/3\,\pi\,R^3\,d)/R^2 = G\,m\,(4/3\,\pi\,d)\,R$ = constants *x* R. So the weight of a mass m = 1 kg on different planets will depend only on the radius R of those planets.

Falling to Earth

Now let's talk about everyday gravity, i.e. the gravity that you feel when you are near the surface of the Earth. When you jump off a diving board (or a bungee tower) gravity pulls you downwards. The force of gravity acts on all parts of your body, and makes them fall faster and faster. You accelerate, but in a very remarkable way: every second that you fall, you pick up an additional 9.8 m/s of velocity. Put in the form of an equation, your velocity v after a time t is

$$v = g\,t$$

where the constant $g = 9.8$ is usually called the *acceleration of gravity*.

By *acceleration* we mean the rate at which your velocity changes. For gravity, the acceleration is constant. After 1 s, your velocity is 9.8 m/s. After 2 s, it is 19.6 m/s. After 3 s, it is 29.4 m/s. After 4 s, it is 39.2 m/s. Every second you pick up an additional 9.8 m/s.

Here is a useful conversion factor:

1 m/s = 2.24 mi/h

To convert meters per second to miles per hour, just multiply by 2.24. So the velocity after 4 s is 39.2 *x* 2.24 = 88 mi/h. That may give you a better sense of how fast you're moving. How far will you fall in a given time? The equation can be derived using calculus.[10] The answer is:

$$D = \frac{1}{2} g\, t^2$$

You are not required to memorize this equation; I show it so you can see how we do these calculations. If we put in $t = 4$ s, this gives $D = 78$ m = 257 ft.[11] That's how far you'll fall in 4 s.

We can also use the equations backwards to see how long it takes to fall a certain distance. In 1939, when King Kong fell off the Empire State Building, it was 330 m high.[12] So how long should it have taken him to fall from the top to the bottom? Take our equation for D and solve for t. This gives:

[10] Optional footnote: According to calculus, the distance $D = \int v\, dt = \int g\, t\, dt = g \int t\, dt = (1/2)\, g\, t^2$.

[11] $D = (1/2)\, 9.8\, (4)^2 = 78$ m

[12] In 1939 the Empire State Building was only 6 years old, and did not yet have the 250-foot TV antenna on its top.

$$t = \sqrt{\frac{2D}{g}}$$

Put in D = 330, g = 9.8, to get t = 8.2 s. Watch the movie (the 1939 version) and see if this matches the time in the movie.

How fast was Kong moving when he hit the ground? We just calculated that t = 8.2 s. We can put that into our velocity equation to get

$$v = g\,t = 9.8 \; x \; 8.2 = 80 \text{ m/s}$$

Remember that 1 m/s is 2.24 mi/h. That means that Kong was falling at 2.24 x 80 = 180 mi/h when he hit. No wonder he was killed.

Dropping Food, Terminal Velocity, and Parachutes

In 2002, during the U.S. war in Afghanistan, food was dropped out of airplanes to feed the Afghan people. The food was dropped from an altitude of 3000 m (about 10,000 ft). How fast was it going when it hit the ground?

Let's use the equations to figure this out. The time it took to fall was:

$$t = \sqrt{\frac{2D}{g}} = \sqrt{\frac{2x300}{9.8}} = 24.7 \text{ s}$$

So its velocity would have been

$$\begin{aligned} v &= g\,t \\ &= 9.8 \text{ x } 24.7 \\ &= 2425 \text{ m/s} \\ &= 5335 \text{ mi/h} \end{aligned}$$

That is incredibly fast. The speed of sound is only 330 m/s, so according to these equations it was going 7.3 times the speed of sound, also called *Mach 7.3*.

Suppose the object slows down by friction with the air, and that all its energy goes into heating the object. (We'll ignore for a moment the heating of the air.) In Chapter 2 we showed that the temperature it reaches at Mach number M will be M^2 x 300 K. So from friction with the air, the food should have heated up to 7.3^2 x 300 K = 16,000 K--that is more than twice as hot as the surface of the Sun! It all should have been vaporized. Clearly that didn't happen to the food dropped on Afghanistan. Our calculation was wrong. But what did we do wrong?

We made the mistake of neglecting the force of air as the packages were dropped. Every time the package hits a molecule of air, it transfers some of its energy. The force *F* of air depends on the area *A* of the package (more area means it hits more air molecules). That's not surprising. But more interesting is the fact that the force depends on the square of the velocity. Double your velocity, and the force goes up by a factor of 4; go 3 times faster, and your force increase by a factor of 9. This means that air resistance becomes extremely important at high velocities. This fact is the key to understanding not only dropped food, but also parachutes, space capsule reentry, and automobile gasoline efficiency.

As the object falls faster and faster, the force of air resistance gets greater and greater. Gravity is pulling down, but the air is pushing up. The force of the air resists the gravity; it opposes it. If the object keeps on accelerating, eventually the force of air will match the weight of the object. When that happens, gravity and air resistance are balanced. The object doesn't stop moving, but it stops accelerating, that is, it no longer gains additional velocity. When this happens we say the falling object has reached *terminal velocity*.

For the food dropped over Afghanistan, the terminal velocity was about 9 mi/h. For a falling person it is typically 70-100 mi/h. That's fast, but people have survived falls from great heights into water. (Try to imagine this next time you are going in a car or train at 74 mi/h.) If a falling person spreads out his arms and legs (like sky divers) that increases the effective area, and the person will fall even slower. A person using a parachute is not much heavier, but the parachute area *A* is large, so the terminal velocity is only about 15 mi/h. For King Kong (about the same area as a parachute, but much heavier) the terminal velocity was several hundred miles per hour; he hit the ground without ever reaching terminal velocity.

It is interesting to think about why objects that fall a long distance will eventually fall at the terminal velocity. If the falling object went any faster, the upward force from the air would be greater than the weight, and that net force would slow the fall. If the object fell any slower than this terminal velocity, then the downward pull of gravity would be stronger than the upward force of air resistance, and the object would fall faster. Only when the velocity reaches the terminal velocity does the object cease to accelerate (go faster or slower).

Here are the terminal velocities for some interesting objects:[13]

food packet:	4 m/s	= 9 mi/h
person in free fall:	33 m/s	= 74 mi/h
person with parachute (larger area):	6 m/s	= 13 mi/h

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[13] Optional footnote: The equation for terminal velocity is $v = \sqrt{\dfrac{2mg}{A\rho}}$. So, for example, if a food packet has a mass of 0.1 kg, an area of 0.1 square meters, and we put in $g = 9.8$ and density $\rho = 1.25$ kg/m^2, this gives $v = 4$ m/s = 9 mi/h.

Automobile Fuel Efficiency

A moving car feels the force of air on its front, and that tends to slow the car down. To keep going at the same velocity, the engine must make up the lost energy. We'll show that much of the gasoline used by the car is to overcome the force of this air resistance. That is an important fact to know. At a velocity of 30 m/s = 67 mi/h, the force of air on the front of the car will be about 500 lb!

To keep the car from being slowed down by this force, the engine must exert an equal and opposite force. That takes a lot of gasoline. At high velocities, more than 50% of the gasoline is used to overcome this air resistance. As a result, car designers have worked hard to "streamline" the shape of automobiles. If the front surface of the car is tilted (rather than flat, as in the old autos from the 1920s) then the force of the air resistance is reduced. On a tilted surface, air molecules can bounce off obliquely instead of hitting the front and bouncing straight back. In such a car, the force can be as low as 100 lb.

Many truck drivers are in business for themselves, and they have to pay for the extra gasoline used to overcome air resistance. Maybe you've noticed the smooth curves that some truck drivers have added to the cabs of their trucks to minimize air resistance. Reducing this force can save substantial money on gasoline.

Figure: Aerodynamic design for a truck cab

The top of the cab has had a contoured shape added to it (called a "cab fairing") to make the air bounce off smoothly, at an angle, instead of hitting the flat face of the truck head on. This type of alteration is sometimes called "aerodynamic smoothing" and it saves gasoline. It reduces the effective value of A, the area in the air resistance equation. Note also that you save even more gasoline by driving slower. At 1/2 the speed, the force is 4 times less. That means the vehicle uses 4 times less gasoline to overcome air resistance.

Force and Acceleration

If you push on an object, and there is no friction to hold it back, then it gains velocity. If you push twice as hard, it gains twice as much velocity. The acceleration is proportional to the force. But it also depends on the mass of the object. If the object has twice the mass, it needs twice the force to get it moving. These two facts are summarized the following equation called **Newton's second law**:

$$F = k\, m\, a$$

In this equation, m is the mass (in kg), a is the acceleration (in m/s), and k is a constant. For the force F to be in pounds, k is 0.224. If you prefer the force in the physicist units called *Newtons* (N)[14], then $k = 1$. That's why Newton's second law is usually written in physics texts as $F = m\, a$.

This equation tells you how much things speed up--or slow down--when you apply a force.

You may recall that Newton's *first* law states that unless an object has an outside force on it, it will tend to keep its motion unchanged. But that is just a special case of the second law, since if $F = 0$, there is no acceleration, and that means no change in velocity. Nobody would ever teach the first law these days if not for the fact that students sometimes wonder what came before the second law.

"g"

In this book, accelerations are typically measured in meters/second per second (m/s^2). For automobiles, we frequently measure acceleration in miles per hour every second (sometimes written as mph/s). For example, if a car salesman tells you that a car will "go from zero to sixty in ten seconds" what he is really telling you is the acceleration: 60 mph in 10 seconds. That is the same as 6 mph every second = 6 mph/s.

[14] Optional footnote: Most physics texts measure force in *Newtons*, so the equation you'll see there is $F = m\, a$. Some physics texts will take this equation as the definition of mass. In that case, Newton's second law can be stated as follows: "the mass of an accelerating object is approximately constant." As we will see in the chapter on relativity, at extremely high velocities, this law breaks down and mass increases with velocity.

Another very useful unit for acceleration, used in the military and by NASA, is the "*g*", pronounced "gee" or "gees" for the plural. One *g* is 9.8 m/s every second, i.e. it is the acceleration of gravity. You've probably heard this term used in news programs and in movies. According to a NASA Web site, when the Space Shuttle is launched, the acceleration reaches a maximum of 3 *g*, pronounced "three gees." A pilot in an F-18 fighter plane sometimes accelerates as much as 10 *g*, "ten gees." Ten *g* is equivalent to 10 x 9.8 = 98 m/s every second. Weightlessness is often described as "zero *g*." That's when there is no gravity, or (if you are in space) no acceleration.

Suppose an automobile is accelerating at 1 *g*. How fast will it be going in 10 s? The answer is found from our velocity equation by setting $a = 9.8$.

$$\begin{aligned} v &= a\,t \\ &= 9.8 \times 10 \\ &= 98 \text{ m/s} \end{aligned}$$

We can convert this to miles per hour by multiplying by 2.24 to get 219 mi/h. Obviously cars don't go this fast. That's because they don't accelerate at 1 *g*, but only at a small fraction of a *g*.

The "*g*-rule"

There is a very good reason to think of acceleration in terms of *g*: it enables you to solve important physics problems in your head. Suppose I told you that I am going to accelerate you in a horizontal direction by 10 *g*. How much force will that take? The answer is simple: 10 times your weight! When astronauts are accelerated by 3 *g* (the maximum acceleration of the Space Shuttle) the accelerating force must be 3 times their weight. I call this the "*g*-rule": **the force to accelerate an object is equal to the number of *g*'s times the weight**

To get the number of *g*'s, just calculate the acceleration in m/s^2, and divide by 9.8. So, for example, we write that an acceleration of $a = 19.6\ m/s^2 = 2\ g$. This is an acceleration of 2 *g*. To accelerate an object to 2 *g* takes a force equal to twice the weight of that object.

Shooting an Astronaut into Space: The Rail Gun

To go into orbit around the Earth, a satellite must have a velocity of 8 km/s. Why not give it this velocity in a gun? Could we literally "shoot" a satellite or an astronaut into space?

The answer is: you might be able to do this, but the astronaut would be killed by the force required to accelerate him. Let's assume we have a very long gun, an entire kilometer long. If we calculate the required acceleration to shoot the astronaut into space, we get $a = 408\ g$, i.e. the acceleration is 408 times the acceleration of gravity.[15]

That's a lot of g's. Remember the g-rule: the force that it will take to accelerate you this fast will be 408 times your weight. So if you weigh 150 lb, the force that must be applied to you is 408 x 150 = 60,000 lb = 30 tons. That is enough to crush your bones.

Suppose we want to experience no more than 3 g. How long would the gun have to be to get you going at v = 8 km/s? The gun would have to be 1000 km long! That is, of course, ridiculously impractical.

Of course, acceleration over such long distances is not impractical--it's what the Space Shuttle does. It takes off, accelerates at 3 g, and it must go over 1000 km to reach orbital speed. So the Space Shuttle accelerates as if it were in a very long gun, but without a barrel. This process actually takes the Space Shuttle further than 1000 km since 3 g is only the peak acceleration; for most of the flight, the acceleration is less.

Acceleration during airplane take-off

The take-off speed for a commercial airplane is about 160 mi/h. What acceleration is needed to achieve this speed in a 1-kilometer runway? Let me make that question a little more personal. You are sitting in such an airplane. In a few seconds, you will be at the other end of the runway, and you will be moving (along with the airplane) at 160 mi/h. What force does the seat have to apply to your back to accelerate you?

We show in the footnote[16] that this requires the airplane to accelerate at 2.6 m/s^2. We can convert this number to g units by dividing by 9.8, to get that the acceleration is 2.6/9.8 = 0.27 g. So the force that the airplane must push on you to get you going that fast is 0.27 times your weight, i.e. about a quarter of your weight. If you weigh 150 lb, then the push you feel on your back will be about 41 lb. Think of this next time you are on an airplane taking off. Does this number feel about right?

Circular acceleration

Physicists like to define velocity as having magnitude and direction. If your velocity changes, we call it acceleration. But suppose you only change your direction and not the

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[15] Optional calculation: The distance D = 8 km = 8000 m, and the velocity v = 8 km/s = 8000 m/s. The relationship between distance, acceleration, and velocity is $v^2 = 2\ a\ D$. Put in v and D, and solving for a gives a = 4000 m/s^2. To covert this to g's, divide by 9.8 to get a = 4000/9.8 = 408 g.

[16] I convert 160 mi/h to meters per second by dividing by 2.2. So the airplane's final speed is 160/2.2 = 73 m/s. I use the same equation I used in the prior footnote: $v^2 = 2\ a\ D$. Put in v = 73, and D = 1000, for $a = v^2/(2\ D) = 73^2/2000$ = 2.6 m/s^2.

actual number of m/s? We still call that acceleration because many of the equations we've been using still work.[17]

The most important example of this kind of acceleration is when you go in a circle. If the magnitude of your speed is v, and this doesn't change (you keep going the same number of meters per second or miles per hour), and the circle has radius R, then we say that the circular acceleration is

$$a = v^2/R$$

This kind of acceleration is very important for the fighter pilot who is trying to change his direction rapidly. For example, if he is moving at velocity of 1000 mi/h, and is turning in a circle of radius R = 2 km, then his acceleration turns out to be 10 g. That's about as much as a fighter pilot can tolerate. We do the calculation in an optional footnote.[18]

The problem with accelerating more than 10 g is that a human's blood pressure is not great enough to keep the blood in the brain, and that causes the pilot to faint. Even specially chosen and physically fit fighter pilots cannot take more than 10 g.

Fighter pilots and astronauts are usually tested and trained in a spinning cylinder. They are rotated fast enough for them to experience a circular acceleration of 10 g.

High g, from circular acceleration, is also a method used to separate the components of uranium for making a nuclear weapon. Such a device is called a *centrifuge*. The heavier parts of the uranium feel a greater force, and they are pulled more strongly towards the outer parts of the spinning cylinder. Such centrifuges are mentioned often in the news. In 2004, Libya disclosed the fact that it had purchased large centrifuge plants for the production of nuclear weapons material. Parts of such centrifuges have been found hidden in Iraq. We'll talk more about these centrifuges when we get to the chapter on nuclear weapons.

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[17] Optional: For people who have studied vectors: velocity is defined in physics as a vector. If the velocity at time t_1 is $\mathbf{v_1}$, and at t_2 is $\mathbf{v_2}$, then the acceleration vector is defined as $(\mathbf{v_2}-\mathbf{v_1})/(t_2-t_1)$. Even if the velocity is only changing in direction, the difference vector $(\mathbf{v_2}-\mathbf{v_1})$ is not zero. For circular motion, its magnitude is given by the equation in the text.

[18] Optional calculation: acceleration of fighter pilot. First we convert to metric units. We convert miles per hour to meters per second by dividing by 2.24, giving v = 1000/2.24 = 446 m/s. We also convert R to meters: R = 2000 m. Plugging in these values gives: a = (446)^2/2000 = 100 m/s^2. We convert this to g'*s* by dividing by 9.8 to get a = 100/9.8 ≈ 10 g

Gravity in Space--According to Science Fiction

Science fiction movies sometimes show a large rotating satellite. There is then "artificial gravity" from the rotation, and astronauts can walk around on the outside. This actually makes sense. The astronauts will feel a force on their feet (which point away from the center of rotation) that will appear to them indistinguishable from gravity. Such a satellite is shown in the classic movie *2001: A Space Odyssey* (1968). We'll show in an optional footnote that the edge of a satellite with a 200-meter radius must be moving at about 44 m/s[19] . That means it will rotate once every 40 s.

Artificial gravity in space--without circular motion

Many science fiction movies show space voyagers in the spaceships walking around as if on the Earth. Is that nonsense? Where does that gravity come from?

I can make sense out of it. I just assume that the spaceships are not moving at constant speed. If the ship engine accelerates the ship by $a = 9.8$ m/s, then the ship will put a force on any astronauts inside to accelerate them too. A person of mass m will feel a force $F = ma$. But since the engines have set $a \approx g$, the force on an astronaut will be $F = m\,g$. That is exactly his Earth weight. If he places his feet on the backside of the ship (opposite in direction to the path the ship is taking) then he will feel like he is standing on Earth. In fact, he can't even tell the difference; the acceleration serves as a "virtual" gravity.[20]

Here is an interesting number: the distance a spaceship would travel in a year if it had constant acceleration of g. I do the calculation in an optional footnote.[21] The answer is 5 $\times 10^{15}$ m, which is about a half light year (the distance that light travels in a year). The distance to the nearest star (not counting the sun) is about 4 light years.

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[19] If the satellite has radius R and is rotating with a rim velocity v, then the force on the astronauts' feet will be $F = m\,a$. If we use the formula for circular acceleration, $a = v^2/R$, then this becomes: $F = m\,v^2\,R$. To make this equal to the astronauts' weight, we set F = mg. Solving for v, we get v = sqrt$(g\,R)$. For a satellite with a radius of R = 200 meters, the rim velocity must be v = sqrt(9.8 x 200) = 31 m/s.

[20] Movies sometimes show the virtual gravity as being off to the side. This could be accomplished by having special sideways thrusters. Every hour or so the thrusters could rotate the ship around so it doesn't wind up going too far sideways.

[21] Optional calculation: the distance that you travel at acceleration g is given by our distance equation $D = (1/2)\,g\,t^2$. We put in $g = 9.8$. For standard physics units (MKS, or meters, kilograms, and seconds) we need to have t in seconds. We can calculate the number of seconds in a year as follows: 1 year = 365 days = 365 x 24 hours = 365 x 24 x 60 minutes = 365 x 24 x 60 x 60 seconds = 3.16 x10^7 seconds. Plugging in this value we finally get D = (1/2) x 9.8 x (3.16 x$10^7)^2$ = 5 x10^{15} m. One light year is the speed of light (3 x10^8 m/s) multiplied by the number of seconds in a year (which we just showed was 3.16 x10^7 s). That gives 9.5 x10^{15} m.

Escape to Space

Suppose you want to completely leave the Earth. Just going into orbit isn't enough; you want to get far away, maybe take a trip to the Moon, Mars, or a distant star. That takes more energy than just going into orbit. How much more? The answer is surprisingly simple: exactly twice as much. You can get that much kinetic energy by going 1.414 times faster than orbital speed. (That's because energy depends on the square of the velocity, and $1.414^2 = 2$.) Since orbital velocity is 8 km/s, the escape velocity is 8 *x* 1.414 ≈ 11 km/s.

These are good numbers to know:

orbital velocity:	8 km/s	for low-Earth orbit (LEO)
escape velocity:	11 km/s	for the Moon or beyond (to infinity!)

It is very interesting that the escape velocity does not depend on the mass. More massive objects do require more energy to escape, but at the same velocity, more massive objects already have more energy! Remember $E = (1/2)\, m\, v^2$. At escape velocity, an object with a mass of 2 kg has twice as much energy as an object with a mass of 1 kg.

I show in the optional footnote[22] that the energy it take to accelerate one gram to escape velocity is 60,500 joules = 14 Calories.

Black Holes[23]

Big planets have high escape velocities. For Jupiter, it is 61 km/s. For the Sun it is 617 km/s. Are there any objects for which the escape velocity is higher than the speed of light, 3×10^5 km/s = 3×10^8 m/s? The surprising answer is *yes*. We call such objects black holes. They get their name from the fact that even light cannot escape, so we can never see their surface; they are black. We'll show in Chapter 11 that no ordinary object (made out of mass as we know it) can go faster than the speed of light. That means that nothing could escape a black hole.

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[22] The mass is 1 gram = 0.001 kg. The velocity v = 11 km/sec = 11000 m/sec. Plug in these values: KE = ½ m v^2 = 0.5 x 0.001 x $(11000)^2$ = 60,500 joules. Divide by 4200 joules per Calorie to get KE = 14 Calories.

[23] Why is the physics of black holes included in a text titled *Physics for future Presidents*? Is there any practical use for this knowledge? The answer is: no, not really. It is included, as are a few other things in this book, just because most people have heard about them and are curious. But you never know. Knowing the size of a black hole once won the author of this book a free guide to Paris. Outside of Shakespeare Books on the west bank of the Seine was a sign offering this prize if anyone could answer the question, "What size would the Earth have to be for it to be a black hole?"

If the black hole is invisible, how do we know it's there? The answer is that even though it can't be seen, we can see the effects of its very strong gravity. Even when there is nothing visible, the gravity of the black hole is so strong that we know there must be something of great mass present, so we deduce it must be a black hole.

To be a black hole, you have to do one of two things: either have a lot of mass, or pack a moderate amount of mass into a very small radius. Several black holes are known to exist; even though we can't see them (no light leaves the surface) we know that they are there because of the strong gravitational force they exert. The known black holes are all as massive as a star or greater. But if you were to take the mass of the Earth and pack it inside a golf ball (in principle, this is possible), then it would be a black hole. The mass would be the same, but the radius would be so small that the gravitational force on the surface of the golf ball would be enormous.

The sun is more massive, so you don't have to pack it so tightly. It would be a black hole if you packed it inside a sphere with a radius of 2 mi.

An interesting fact is that the force of gravity on an object of mass m (e.g. you) exerted by a spherical object of mass M (e.g. the Sun) is given by $F = GMm/r^2$. (No, you don't need to know this equation.) Notice that the size of the object (the Sun) doesn't enter the equation. That means that even if the Sun were compressed into a black hole, its gravity at the distance of the Earth wouldn't change, because its mass M didn't change.

Of course, if the Sun became a black hole, it would have a tiny radius of only 2 miles. The gravity on the surface of the black-hole sun would be much greater than the gravity on the surface of the present sun, because r is so much smaller. That's why black holes have a deserved reputation for strong gravity: they are so small that you can get very close to a large amount of mass.

There are several black holes that were created, we believe, when the inner part of a star collapsed from its own weight into a very small radius. The object in the sky known as Cygnus X-1 is thought to be a black hole from such a collapse. If you have spare time, look up Cygnus X-1 on the Internet and see what you find.

Many people now believe that large black holes exist at the center of the large collections of billions of stars known as galaxies, such as the Milky Way galaxy. We presume these were created when the galaxy formed, but we know almost none of the details about how this happened.

Even more remarkably, the Universe itself may be a black hole. That's because the black hole radius for the Universe is about 15 billion light years, and that is approximately the size of the observable Universe. In other words, the Universe appears to satisfy the black-hole equation. We'll discuss this further in Chapter 12. But you probably won't be surprised at one inescapable[24] conclusion: we can never escape from the Universe.

[24] pun intended

Momentum--Return to Newton's Third Law

If you shoot a powerful rifle, then the rifle puts a large force on the bullet sending it forward. But the bullet puts a backward force on the rifle, and that is what causes the "kick." The rifle can suddenly go backwards so rapidly that it can hurt your shoulder. If you don't have your feet firmly planted on the ground, you will be thrown backwards.

Recall that we discussed Newton's third law earlier--the fact that if you push on something, it pushes back on you. Newton stated this as "for every action there is an equal and opposite reaction." But we no longer use this old terminology of "action" and "reaction." Instead we say that if you push on an object (such as a bullet), then the bullet also pushes back on you for exactly the same amount of time. Of course, the bullet is lighter, so it is accelerated much more than you are.

Based on the fact that the bullet pushes on the rifle for the same time that the rifle pushes on the bullet, we can derive an extremely important equation, sometimes called the conservation of momentum.[25] For the rifle (subscripts *R*) and the bullet (subscripts *B*), the equations are

$$m_B \, v_B = m_R \, v_R$$

The equation for the rifle recoil is simple: the mass of the bullet, times its velocity, is the same as the mass of the gun times its recoil velocity. Of course, the velocities are opposite; this is sometimes indicated by putting a minus sign in front of one of the velocities. (I didn't do that here.) When the gun is stopped by your shoulder, then you recoil too--but less, because you have more mass.

The product $m\,v$ is called the *momentum*. One of the most useful laws of physics is called the *conservation of momentum*. Before the rifle was fired, the bullet and gun were at rest; they had no momentum. After the rifle fired, the bullet and the rifle were moving in opposite directions, with exactly opposite momenta (the plural of momentum), so the total momentum was still zero.

Here's another way to say that: when you fire a gun, the bullet gets momentum. You and the rifle you are holding get an equal and opposite momentum. If you are braced on the ground, then it is the Earth that recoils with that momentum. Because the Earth has large mass, its recoil velocity is tiny and difficult to measure.

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[25] The derivation is based on calculus. If an object is at rest, and it experiences a force F for a time t, then its velocity $v = \int a\, dt = \int (F/m)\, dt$. Write this as $m\,v = \int F\, dt$. If there are two objects, and the forces are equal and opposite, and the time is exactly the same, then $\int F\, dt$ is equal and opposite for the two objects, and that means that the quantity $m\,v$ must also be equal and opposite for the two objects.

If the objects are in motion prior to the force acting, then the *changes* in momentum must be equal and opposite. Let's apply that to the comet that crashed into the Earth and killed the dinosaurs. To make the calculations easy, assume that before the collision the comet with mass m_C was moving at v_C = 30 km/s (a typical velocity for objects moving around the Sun). Assume the Earth was at rest. After the collision, the total momentum would be the same. The Earth would have mass m_E (that now included the mass of the comet) and have velocity v_E. So we can write

$$m_C\, v_C = m_E\, v_E$$

Solving for v_E we get

$$v_E = m_C\, v_C / m_E$$

The mass of the comet is about 10^{19} kg.[26] Everything else is known, so we can plug into this equation and get the v_E, the velocity of recoil of the Earth:

$$\begin{aligned} v_E \quad &= (10^{19})(30000)/(6\text{x}10^{24}) \\ &= 0.05 \text{ m/s} \\ &= 5 \text{ cm/s} \\ &= 2 \text{ in/s} \end{aligned}$$

That's not much of a recoil, at least when compared to the usual velocity of the Earth, which is 30 km/s = 30,000 m/s. So the Earth was hardly deflected. It's orbit changed, but only by a tiny amount.

Let me estimate how much a truck recoils when it is hit by a mosquito. Assume, from the point of view of the truck, that a mosquito with mass m = 2.6 mg is moving at 60 mi/h = 27 m/s. Assume the truck weighs 5 metric tons = 5000 kg. (Hint: use the same equation, except let the 2.6-milligram mosquito represent the comet. Don't forget to convert everything to the same units--kilograms and meters.)

Although the conservation of momentum is one of the most important laws of physics, it is violated in many action movies. For example, if the hero in *The Matrix* (1999) punches the villain, and the villain goes flying across the room, then the hero should go flying backwards (unless he is braced on something big and massive). Likewise, small bullets, when they hit a person, seem able to impart very large velocities to the person that they hit, so the person goes flying backwards.[27]

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[26] A comet with a radius R_C = 100 km = 10^5 m has a volume of $V_C = 4/3\, \pi R_C^{\,3}$ (for a sphere) = $4.2\text{x}10^{15}$ cubic meters (m^3). Assuming that the comet is made mostly of rock and ice, the density is probably about 2500 kg/ m^3, so the mass is 2500 x $4.2\text{x}10^{15}$ = 10^{19} kg.

[27] As a fan of this movie, I explain to myself that according to the script, "reality" is just a computer program called *the matrix*. Therefore I can assume that whoever programmed *the matrix* simply forgot to put in conservation of momentum. (Or, as James Gill suggests, Neo and the others altered the program to violate the laws of physics.)

Rockets

Imagine trying to get into space by pointing a gun downward and firing bullets so rapidly that the recoil pushes you upward. Sound ridiculous? Yet that is exactly how rockets work.

Rockets fly by pushing burned fuel downward. If the fuel has mass m_F and is pushed down with a velocity v_F, then the rocket (which has mass m_R) will gain an extra upward velocity v_R given by the same kind of equation we used for the rifle:

$$v_R = v_F\, m_F/m_R$$

Compare this to the rifle equation, and to the comet/Earth collision equation.

Because the rocket weighs so much more than the fuel which is expelled every second (i.e. m_F/m_R is tiny), the amount of velocity gained by the rocket is much less than the fuel velocity. As a consequence, rockets are a very inefficient way to gain velocity. We use them to go into space only because in space there is nothing to push against except expelled fuel. Another way to think of this is as follows: rockets are inefficient because so much of the energy goes into the kinetic energy and heat of the expelled fuel, rather than in the kinetic energy of the rocket.

The equation above gives the velocity *change* when a small amount of fuel is burned and expelled. To get the total velocity given the rocket, you have to add up a large number of such expulsions. Meanwhile, the mass of the rocket (which is carrying the unused fuel) is changing as fuel is used up. A typical result is that the rocket must carry huge amounts of fuel. The mass of fuel used is usually 25 to 50 times larger than the payload put into orbit.

For a long time, this huge fuel-to-payload ratio led people to believe that shooting rockets into space was impossible; after all, how could you even hold the fuel if it weighed 24 times as much as the rocket? The problem was solved by using rockets with multiple stages, so that the heavy containers that held the initial fuel never had to be accelerated to the final orbital velocity. For example, the Space Shuttle has a final payload (including orbiter weight) of 68,000 kg = 68 tons (T), but the boosters and the fuel weigh 1,931 T, a factor of 28 times larger.[28] Of course, the booster never gets into orbit, only the much smaller shuttle.

Balloons, rockets, and astronaut sneezes

When you inflate a balloon, and then release the end, the balloon goes whizzing around the room, driven by the air being pushed out the end. What is happening is nearly

[28] The external tank holds 751 T of fuel, and there are two solid rocket boosters that weigh 590 T each, for a total of 1931 T.

identical to the way a rocket flies. Before you release the balloon opening, the balloon and the air have zero total momentum. When you let the air come out, it rushes out with high speed, pushed by the compressed air in the balloon, and it pushes back. It pushes back on that air, and that air pushes on the balloon. The released air goes one way, and the balloon containing its remaining air goes the other way, and the two momenta cancel.

When you sneeze, the sudden rush of air outward likewise can push your head backwards. In the opening of this chapter, there was a puzzle: how much does the astronaut's head snap back? I read in a newspaper once that the head would snap back at a dangerous speed because the head was weightless in space. That's not true, of course. The writer of the article had confused weight with mass. The astronaut's head has no weight but it has every bit as much mass as it did on Earth. The force of the sneeze will accelerate the head backwards by an amount given by $F = k\, m\, a$, but since m is the same as on Earth, the acceleration a will be no greater.

Skyhook

Ponder the Space Shuttle. To put a 1-gram payload into orbit requires 28 grams of extra weight (fuel + container + rocket). For a rocket that is going at escape velocity, the efficiency is even worse.[29] But let's assume this number, and compare it to the energy it would take to lift an object to the same altitude. Suppose we build a tower with an elevator that goes all the way up to space.[30] How much energy would it take to haul the gram up to the top, using the elevator? According to the section "Escape to Space," the energy required to take a gram of material to infinity is 15 Cal. That's the energy in 1.5 grams of gasoline (not including the air). So using a rocket takes 28/1.5 = 19 times more fuel than using an elevator.

Many people have pondered the fuel waste from rockets. Although a tower to space seems impossible, it may make sense to hang a cable down from a geosynchronous satellite and use it to haul payload up, an idea once referred to as "project skyhook." The recent discovery of very strong carbon nanotubes has revived the idea. Arthur C. Clarke used this idea in his 1977 science fiction novel *Fountains of Paradise*.

A more likely idea is to "fly" to space on an airplane. Airplanes have two attractive features: they use oxygen from the atmosphere as part of their fuel (so they don't have to carry it all, as do rockets) and they can push against the air, instead of having to push against their own exhaust. Although it is possible in principle, the technology to achieve

[29] That's because the rocket has to go much faster than its spent fuel is ejected, so that this fuel has high velocity in the upward direction. It's kinetic energy is wasted.

[30] In the Bible, such a tower was attempted in ancient Babylon, and it is also referred to as the "Tower of Babylon." Its goal was to reach heaven. To prevent the Babylonians from succeeding, God made all the workers speak different languages. Thus, according to the Bible, this is the origin of the multitude of languages spoken by humans. It is also the origin of the verb "to babble."

8 to 12 km/s with airplanes does not yet exist. We'll talk more about airplanes in a moment.

Earlier in this chapter we mentioned the possibility of using rail guns. These are long devices that achieve their high projectile velocities by using electric and magnetic forces to push on the projectile. But recall their limitation: they must be very long in order to avoid huge accelerations and the huge g-forces that would result. Even trained fighter pilots can only endure about 10 gs.

Ion Rockets

The inefficiency of rockets comes from the fact that typical chemical fuels only have enough energy to give their atoms a velocity of 2-3 km/s. Rockets could possibly overcome this limitation by shooting out *ions*, a name for atoms that have an electric charge. You can find out a lot about these on the Internet. Like the rail gun, the ions can be given their high velocity through electric forces, so they are not limited to the velocity of 2-3 km/s that is typical of rocket fuel. For example, a proton expelled at an energy of 100,000 eV has a velocity of 4400 km/s. This makes ion rockets potentially much more efficient than chemical rockets, but so far nobody has figured out how to make the mass of the expelled ions sufficiently great to be able to launch a rocket from the Earth. They are more useful when a low thrust is needed for an extended period of time. NASA is planning on using ion propulsion on the upcoming Dawn Mission to the asteroids Vesta and Ceres (possibly launched as early as 2006).

Flying: Airplanes, Helicopters, and Fans

Airplanes fly by pushing air downwards.[31] Every second, the airplane tends to pick up downward velocity from the Earth's gravity. It stays at the same altitude by pushing enough air downward that it overcomes this velocity.

The fact that wings push air down is most readily observed in a rotary-wing aircraft, otherwise known as a helicopter. (Helicopter pilots call the ordinary airplane a "fixed-wing" aircraft.) In fact, the helicopter blades are designed to have a shape identical to wings, and air is pushed past them when they spin. They push the air down, and that is

[31] In most physics books, the lift on the airplane wing is explained by use of a principle called Bernoulli's law. The "derivation" is done using a diagram that typically shows the air trailing the wing as if it is completely undisturbed. The astute student will be bothered by this. How can the air put a force on the wing but the wing not put a force on the air? A careful analysis (done in advanced aerodynamics books) shows that to establish a flow with higher velocity above the wing than below, the air velocity distribution far from the plane is not undisturbed; in fact, it has air deflected downward with a momentum rate equal to the upward force on the wings, as it must to satisfy momentum conservation.

what pushes the helicopter upward. If you stand under a helicopter rotor when it is spinning, you can feel the air being forced downward.

It is probably more convenient, however, to observe how wing-shaped blades push air by standing in front of a fan. Fans work the same way as a helicopter blades, and as airplane wings. The blade's movement through the air pushes air perpendicular to the direction of motion of the blade.

For the airplane and the rocket, the v_R needed is the velocity to overcome the pull of gravity. In one second of falling, gravity will give any object a velocity of

$$v = g\,t \approx 10 \text{ m/s}$$

This falling velocity must be cancelled by accelerating upward, and this is done in an airplane by pushing air downwards. Air is typically a thousand times less dense than the airplane (1.25 kg/m^3), so to get enough air (i.e. to make the mass of the air large) the wings must deflect a large amount of air downward.

The wake of a large airplane consists of this downward flowing air, often in turbulent motion. It can be very dangerous for a second plane if it encounters this wake, since the amount of air flowing downward is large.

Flying: Balloons

The first way that humans "flew" was in hot air balloons, in 1783 above Paris. These make use of the fact that hot air expands and it takes more volume compared to an equal mass of cool air. Another way to say this is that the density of hot air (the mass per volume) is less than that of cool air.

In a liquid or gas, things that are less dense tend to float. That's why wood floats on water (if it has a density less than 1 g/ cm^3; some woods sink). The heavier fluid "falls" and flows under the less dense object, pushing it upward. Boats float only if their average density (metal hull plus empty space inside) is less than that of water. That's why a boat will sink if it fills with water.

If you fill a balloon with hot air, it will rise until it reaches an altitude at which the density of the surrounding air matches that of the balloon. (Of course, you have to include the mass of the balloon, and any weight that it carries, along with the mass of the air inside.)

Better than hot air are light gases such as hydrogen and helium. Air weighs about 1.25 kg/m^3 (at sea level). The same volume of hydrogen weighs about 14 times less, i.e. about

0.089 kg = 89 g.[32] If we fill a 1-cubic-meter balloon with hydrogen, it will tend to float. Calculate how much 1 m^3 of air weighs, and how much 1 m^3 of hydrogen weighs. The "lift" (upward force) of the balloon is just equal to the difference between these two weights. Putting this into numbers, it means that a cubic meter balloon will have a lift of 1.25 – 0.089 = 1.16 kg. That means that if you hang an object from it that weighs less than one kilogram (including the mass of the balloon skin), it will still go upwards.

Helium gas isn't quite as light as hydrogen gas, so if the balloon is filled with helium, the lift isn't quite as much. Here's the calculation: helium gas weighs about 0.178 kg per cubic meter, so the lift on a helium balloon would be 1.25 – 0.178 = 1.07 kg. Note that even though the helium is twice as dense as hydrogen, the lift is almost as good.

But 1.16 kg of lift for 1 m^3 of balloon is not really much. That's why, despite the cartoons you may have watched on TV, even a large packet of balloons are not enough to lift a 25 kg child. (If you have some spare time, and want to be amused, look up "Lawn-chair Larry" on the Internet.)

Hot air balloons have even less lift. If you heat the air to 300 C, then its temperature in absolute scale is 600 K. That's twice its normal temperature, so its density is half of its usual density. The lift of a cubic meter would be 1.25 – 1.25/2 = 0.62 kg/m^3. Notice that the lift is significantly worse than hydrogen or helium. To lift a person who weighs 100 kg (including basket, balloon skin, and cables) would take 100/0.62 = 161 m^3 of hot air. If the balloon were shaped like a cube, the side of such a balloon would be $\sqrt[3]{161}$ = 5.4 meters = 18 feet. That's why hot air balloons have to be so big, and why they don't lift much.

Floating on water

The same principles apply to floating on water. Salt water is denser than fresh water, so the density difference between it and you is greater; that's why you float higher in salt water. Even strong swimmers take advantage of their buoyancy (the fact that they are less dense, on average, than water). If water becomes filled with bubbles, its average density can become less than yours, and you will sink. The New York Times on August 14, 2003, had an article describing how a group of boys drowned because they were in bubbly water.[33] Undersea volcanic eruptions have led to bubbly water in the oceans, and in such water even ships will sink.

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[32] Nitrogen has an atomic weight (number of neutrons plus protons) of 14. Hydrogen has an atomic weight of 1. The number of atoms in a cubic meter is the same for both gases, so the factor of 14 simply reflects the larger nitrogen nucleus.

[33] Here is a quote from that article:

To four teenagers from the suburbs, Split Rock Falls was a magical place --cool water rushing between the granite walls of a mountain ravine, forming pools for hours of lazy summertime swimming.

On Tuesday afternoon the four men-- Adam Cohen, 19; Jonah Richman, 18; Jordan Satin, 19; and David Altschuler, 18--returned to their favorite childhood summer haunt to

Submarines can adjust their depth below the ocean surface by using ballast tanks to take in or expell water. When they take in water, the air in the tanks is replaced by the heavier water, and the average density of the submarine increases. This makes the submarine sink. The only thing that will stop the sinking is the expulsion of water from the tanks. If the submarine sinks too far, then it is crushed by the weight of the water above it; that makes it even more dense, and so it sinks faster. That is called the hull crush depth. In the movie *Crimson Tide*, the hull crush depth was 1800 ft, about 1/3 mi.

The submarine in that movie (the USS Alabama) was able to save itself by getting its engine running, and using its short "wings" to get lift in a way similar to the way that an airplane does--by moving forward and pushing water downward. A submarine can also push compressed air into its ballast tanks, driving out the water, and decreasing its average density.

Sperm whales are said to be able to dive as deep as 2 mi. Diving deep is easy, since as the whale goes deeper, any air in its lungs (remember, a whale is a mammal) is compressed, and that makes the whale less buoyant. So once the whale is denser than water, it will sink. Coming up is the hard part. Whales save enough energy in their effortless dives, to be able to swim back up to the surface.

Air Pressure on Mountains, Outside Airplanes and Satellites

Air pressure is simply the weight of the air above you. In any fluid or gas, the pressure is evenly distributed, so that the air at sea level will push an equal amount up, down, and sideways. As you go higher, there is less air above you, so the pressure decreases. At an altitude of 18,000 ft (3.4 mi, 5.5 km) the pressure is half of what it is at sea level. That is the altitude of Mt. Kilimanjaro in Africa, and many people walk to the top every year. They go above half the atmosphere! It is hard to breathe up there, and nowhere on Earth do people live continuously at that altitude. But if you only spend a day or two going up and down, you can take it. Go up another 18,000 ft to an altitude of 36,000 ft and the pressure is reduced, by another factor of 2, to one quarter the pressure at sea level. That's

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find it engorged by a summer of heavy rain. By the end of the day, all four men, each an experienced swimmer, was dead, drowned in the waters they knew well.

In what officials here described as one of the worst drowning accidents ever in the Adirondack State Park, all four died after Mr. Altschuler slipped off a narrow granite ledge into a foaming pool of water whipped into a frenzy by a tumbling waterfall. In a final act of friendship, Mr. Richman, Mr. Cohen and Mr. Satin, who had grown up together on Long Island, jumped after him to try to save his life, police and officials said. The laws of physics were against them, though.

"They call it a drowning machine," said Lt. Fred J. Larow, a forest ranger with the State Department of Environmental Conservation, who helped recover the bodies here, about 20 miles east of Lake Placid. "The water was so turbulent and aerated that there was no way they could stay above water. Even the strongest swimmer in the world couldn't have survived it."

a typical altitude for jet airplanes. And that rule continues; for every additional 18,000 ft, the pressure (and the density of the air[34]) drops by another factor of 2. You can not live at such a low density of air, and that is why airplanes are pressurized. You can live at that pressure if the "air" you are given is pure oxygen (rather than 20%), and that is what you would get from the emergency face masks that drop down in airline seats if the cabin ever loses pressure.

Want to know the decrease in air pressure? Divide the altitude by 18,000 ft (or by 5.5 km, if that's how you've measured it). The number you get is the number of halvings you have to do. Suppose your altitude (e.g. in an airplane) is 40,000 ft. Divide that by 18,000 to get about 2 halvings. That means the pressure is reduced by a factor of (1/2)x(1/2) = 1/4. So the pressure (and the air density) is 4 times less than at sea level.

Now consider a "low-Earth orbit" satellite, at H = 200 km above sea level. Let's do all our calculations in kilometers. So we use the half altitude distance to be 5.5 km. Then $H/5.5 = 200/5.5 \approx 36$. Now multiply (1/2) by itself 36 times to give:

$$P = (1/2)^{36} = 1.45 \text{ x}10^{-11} = 0.00000000001$$

That pressure is 10 trillionths times as small as the pressure at sea level.[35] Satellites need this low pressure to avoid being slowed down by collisions with air. A typical altitude for LEO (remember? "Low-Earth Orbit") is 200 km.

Because the density of air decreases with altitude, a helium balloon will not rise forever. Eventually it reaches an altitude at which the outside air has the same density as the helium (with the weight of the gondola averaged in), and then it stops rising. I noticed this as a child, when I was disappointed to see that the helium-filled balloon I had released did not go all the way to space, but went high up and then stopped rising. That's why balloons are not a possible way to get to space.

Space and the X Prize

How high do you have to go to reach space? That depends on your definition. You never get far enough to completely escape the Earth's gravity, and likewise, no matter how high you get, there will always be some atmosphere (even if it's only one molecule per cubic mile). Since the lowest satellite will fly, at least for a few orbits, if it orbits at an altitude of 100 km, some people have defined this altitude as being "space." (Even at 100 km there is enough atmosphere to slow the satellite after a few orbits, and bring it to the surface.)

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[34] The air density doesn't drop quite that much because the air up there is cooler.

[35] The equation implicitly assumed that the temperature of the air continues to cool at higher and higher altitude. But above the tropopause the temperature rises, and that makes this calculation interesting, but not really accurate.

To encourage private companies to send rockets or aircraft into space, a private foundation created the "X Prize" for the first private organization to send a human to that altitude. I'm not going to go into all the details of this prize, because it is far less interesting than most people think. There is a huge difference between reaching a 100-kilometer altitude and orbiting the Earth. Way back in 1946 the U.S. government launched a modified German V-2 rocket to an altitude of 187 km--but it was not until 1957, 11 years later, that the Soviet Union launched the first Earth-orbiting satellite. Getting to orbit is much harder than merely getting into space! Let me show you why.

To get a feel for the relative difficulty of actually going into orbit, imagine that there is NO atmosphere on the Earth. Suppose we want to shoot a bullet straight up, and have it reach an altitude of 100 km. How fast must it leave the gun? The answer is 1.4 km/s. For students who have studied the equations, I put the calculation in the footnote.[36] Compare this to the speed needed for orbit: 8 km/s. Orbital speed is 5.7 times greater than the speed it takes to get up 100 km. Moreover, the kinetic energy depends on the square of the velocity, so the energy needed is a factor of $(5.7)^2 \approx 33$ times bigger. So it takes 33 times as much energy to get into orbit than to merely reach a 100-kilometer altitude.

So why is there so much excitement over the X Prize? I think it is because very few people know about this factor of 33. (Even many professors are surprised.) The winners of the X Prize certainly understand, but they don't want to publicize that. The $10-million prize was not enough to cover their expenses! So how will they make money? Not by sending up satellites. They can't accomplish that any more than the U.S. could with the V-2 rocket. The interesting answer: the winners of the X Prize will earn money by getting tourists to fly up 100 km, and then telling the tourists that they now qualify as "astronauts."

Convection--Thunderstorms and Heaters

When air is heated near the ground, its density is reduced and it tends to rise, just like a hot air balloon. Unconstrained by any balloon, the air expands as it rises, and an interesting result is that it will remain less dense than the surrounding air until it reaches the tropopause, the level at which ozone absorbs sunlight causing the surrounding air to be warmer. When the hot air reaches the tropopause, its density is no longer less than that of the surrounding air, so the air stops rising.

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[36] Optional calculation: speed to reach 100 km. The energy to reach height h is $E = mgh$, where g is the acceleration of gravity, approximately equal to 10 m/s^2. If m = 1 km, and h = 100 km = 1E5 m, then E = 1 x 10 x 1E5 = 1E6 j. The velocity that you need, in order to calculate mass, can be calculated from $E = \frac{1}{2} m v^2$ = 1E6 j. Solving for v gives v = sqrt(2E6) = 1414 m/s = 1.4 km/s.

On a summer day, when thunderstorms are growing, it is easy to spot the tropopause. It is the altitude at which the thunderstorms stop rising, and begin to spread out laterally. The tropopause is a very important layer in the atmosphere. It is the location of the ozone layer, which protects us from cancer-producing ultraviolet light. We'll talk more about this layer in Chapter 7 "Waves" because of the important effect it has on sound, and in Chapter 9 "Invisible Light" when we discuss ultraviolet radiation and its effects.

Convection is the name we give to the process of hot air expanding and rising. When you have a heater in a room near the floor, the rising hot air forces other air out of the way; that results in a circulation of the air. This is a very effective way to warm a room--much faster than heat conduction through the air. But it invariably results in the warmest air being near the top of the room. If you put the heater near the top, the warm air just stays up there. On a cold day, in a room with a heater near the floor, stand up on a stepladder and feel how much hotter it is near the ceiling.

Optional: Angular Momentum and Torque

In addition to ordinary momentum (mass times velocity), there is another kind of momentum that physicists and engineers find enormously useful in their calculations called angular momentum. Angular momentum is similar to ordinary momentum but it is most useful for motion that is circular, i.e. rotation. If an object of mass M is spinning in a circle of radius R, moving at velocity v, then its angular momentum L is

$$L = M\,v\,R$$

What makes angular momentum so useful is that, like ordinary momentum, when there are no external forces on an object, it is "conserved," i.e. its value doesn't change. Have you ever spun on ice skates? Actually, ice skates are not necessary--just stand in a spot and start spinning with your arms stretched out. If you've never done this, then I strongly recommend you try it right now. As you spin, rapidly pull your arms in. (You'll never forget the experience, and you can have great fun entertaining children with it.) To the surprise of most people, they will suddenly spin much faster. You can predict that from the angular momentum equation. If the angular momentum L is the same before and after the arms are pulled in, and the mass of the arms M is the same, then $v\,R$ must be the same. If R gets smaller, then v must get bigger.

Angular momentum conservation also explains why water leaving a tub through a narrow drain begins to spin. In fact, it is very unlikely that the water in the tub wasn't already spinning, at least a little bit. But when the distance to the drain (R, in the equation) gets small, the v in the equation gets very big. A similar effect occurs in hurricanes and tornadoes. Air being sucked into the center (where there is a low pressure due to air moving upward) spins faster and faster, and that's what gives the high velocities of the air in these storms. The air in hurricanes gets its initial spin from the spin of the Earth, and amplifies by the angular momentum effect. So hurricanes really do spin in different directions in the Southern and Northern Hemispheres.

It is not true that sinks or tubs drain differently in the Northern and Southern Hemispheres. Their direction depends on the small residual rotation in the water left over from the filling of the tub, or from a person getting out.

Conservation of angular momentum can be used to understand how a cat, dropped from an upside down position, can still land on his feet. (Don't try this one at home! I never have, but I've seen a movie...) If he spins his legs in a circle, his body will move in the opposite direction, keeping his total angular momentum equal to zero. That way he can bring its legs underneath him. Astronauts do this trick when they want to reorient themselves in space. Spin an arm in a circle, and your body will move in the opposite direction. You can try that trick on ice skates too.

The conservation of angular momentum has other useful applications. It helps keep a bicycle wheel from falling over, when the wheel is spinning. It can also cause a difficulty: if kinetic energy is stored in a spinning wheel (usually called a flywheel), then the angular momentum makes it difficult to change the direction that the wheel is spinning in. This makes the flywheel’s use for energy storage in moving vehicles, such as buses, problematical. It is often addressed by having two flywheels spinning in opposite directions, so although energy is stored, the total angular momentum is zero.

Angular momentum can be changed by a suitable application of an outside force. The required geometry is that the force must act obliquely, at a distance. We define torque as the tangential component of the force (the oblique part) times the distance to the center. So, for example, to start a bicycle wheel spinning, you can't just push on the rim in a radial manner. You have to push tangentially. That's called torque. The law relating torque and angular momentum is as follows: the rate of change of angular momentum is numerically equal to the torque.

You can probably see why mastery of the equations of angular momentum is very useful to engineers and physicists in simplifying their calculations.

END OF CHAPTER

Quick Review

Weight is the force of gravity acting on mass. The force of gravity obeys an inverse square law, so when the distance increases by (for example) 3, the force becomes 9 times weaker. It is this force that keeps the Moon in orbit around the Earth, and the Earth in orbit around the Sun. If gravity were turned off, satellites would move in straight lines rather than in circles. Even at great distances, the force of gravity never goes completely to zero. The sensation of weightlessness felt by astronauts is really the sensation of continuous falling.

All satellites must keep moving, or they fall to Earth; they cannot hover. In LEO, the satellite moves at 8 km/s, and orbits the Earth in 1.5 h. LEO satellites are useful for Earth observations, including spying. Geosynchronous satellites are useful for applications where the position of the satellite must stay fixed with respect to the ground. A medium-Earth orbit (MEO) is in between. GPS satellites are MEO. A GPS receiver determines its location by measuring the distance to 3 or more of these satellites.

Close to the Earth (where we live) objects fall with constant acceleration, i.e. their velocity $v = g\,t$, until they are slowed down by air resistance. The distance they fall is $h = (1/2)\,g\,t^2$. Air resistance increases with the square of the velocity. If you go 3 times faster, the force of air resistance gets 9 times larger. Air resistance limits the velocity of falling to no more than the "terminal velocity." This depends on the area and mass of the object, and is different for people (60-100 mi/h), parachutes (15 mi/h), and large objects such as King Kong. Air resistance also limits the fuel efficiency of automobiles. Satellites must fly high (> 200 km) to avoid air resistance.

A force on an object makes it accelerate by an amount (given in m/s^2) of

$$F = k\,m\,a$$

This is known as Newton's second law ($k = 0.224$ is a constant that makes the units come out in pounds). Acceleration can also be measured in units of g, the acceleration of gravity. The g-rule says that to accelerate an object to 10 g requires a force 10 times as great as that object's weight. That's about the limit for a human to endure, so higher accelerations (e.g. the rail gun) can't be used. The Space Shuttle never accelerates more than 3 g. Circular motion can also be considered acceleration, with Newton's second law applying, even if the magnitude of the speed doesn't change. Based on this, we can calculate the velocity a satellite must have to stay in a circular orbit at different altitudes above the Earth.

The surface gravity on other planets and on asteroids is very different than it is on the Earth, science fiction movies notwithstanding.

To escape to space completely requires an energy of about 15 Cal/g. This is enough energy to lift you up a tall enough elevator, if one could be built (project skyhook). If you have a velocity of 11.2 km/s, then your kinetic energy is sufficient to escape. Black holes are objects whose escape velocity exceeds the speed of light.

Gravity measurements have practical applications. Since oil is lighter than rock, it has a weaker gravity, and that fact has been used to locate it. Gravity measurements give us the best image of the Chicxulub crater.

When a gun is fired, the bullet goes forward and the gun goes backward. This is an example of the conservation of momentum. Other examples: rockets go forward (very inefficiently) by shooting burnt fuel backwards; airplanes and helicopters fly by pushing air downwards.

Objects float when their density is less than that of the fluid or gas they are in. That includes boats and balloons. Hot air rises because it is less dense than the surrounding air, and that happens for hot air balloons and thunderstorms. The density and pressure of air decreases with altitude according to a halving rule

$$P = (1/2)^{H/6.9}$$

where the height H is expressed in km.

Angular momentum (a momentum that applies to circular motion) is also conserved, and that causes contracting objects to speed up. Examples include sink drains, hurricanes, and tornadoes.

Qualitative Questions

At high velocities, most of the fuel used in an automobile is used to overcome
- ( ) gravity
- ( ) momentum
- ( ) air resistance
- ( ) buoyancy

Airplanes fly by
- ( ) pushing fuel downward
- ( ) pushing fuel backwards
- ( ) pushing fuel upwards
- ( ) pushing air downwards

At an altitude of 200 km, the downward force of gravity on an Earth satellite is
- ( ) the same as on the surface of the Earth
- ( ) a little bit weaker
- ( ) zero
- ( ) a little bit stronger

In the Southern Hemisphere, sinks drain
- ( ) usually clockwise
- ( ) the same way they do in the Northern Hemisphere
- ( ) the answer is different in the Eastern and Western Hemispheres.

A rail gun
- ( ) accelerates rails
- ( ) can shoot bullets faster than ordinary guns
- ( ) doesn't really accelerate things
- ( ) uses ions for propulsion

The Sun would be a black hole if it were squeezed into a radius of
- ( ) 2 miles
- ( ) 2,000 miles
- ( ) 2 million miles
- ( ) It already is a black hole

The method that would take the least energy to get something to space is:
- ( ) lift it by elevator (if the elevator existed)
- ( ) launch it in a one-stage rocket
- ( ) fly it in a balloon
- ( ) launch it in a three-stage rocket

If an artificial satellite orbited the Earth at the distance of 240,000 miles (the present distance to the moon), it would orbit the Earth in a period of

( ) 90 minutes
( ) 1 day
( ) 1 week
( ) 1 month

The time it takes a geosynchronous satellite to orbit the Earth is:

( ) 90 minutes
( ) 1 day
( ) 1 week
( ) 1 month

A typical terminal velocity for a high diver is closest to:

( ) 1 mi/h
( ) 10 mi/h
( ) 100 mi/h
( ) 1000 mi/h

The force of gravity between two people standing next to each other is

( ) zero
( ) too small to measure
( ) small but measurable
( ) greater than 1 kg

For a low-Earth satellite (altitude 200 km), the force of gravity is

( ) zero
( ) a few percent less than on the Earth's surface
( ) exactly what it is on the Earth's surface
( ) about half of what it is on the Earth's surface

The altitude of a geosynchronous satellite is closest to:

( ) 200 miles
( ) 26,000 miles
( ) 2,000,000 miles
( ) 93,000,000 miles

Rockets fly by

( ) using antigravity
( ) pushing air downwards
( ) pushing fuel downwards
( ) being lighter than air

Balloons rise until

( ) the surrounding air becomes too dense

( ) the surrounding air becomes too thin (not dense)
( ) the surrounding air becomes too cold
( ) they reach space

You can see where the tropopause is by
( ) looking at Earth satellites
( ) seeing how high birds fly
( ) looking at the tops of thunderstorms
( ) seeing where lightning comes from

Ice-skaters spin faster by pulling in their arms. This illustrates that
( ) momentum is conserved
( ) angular momentum is conserved
( ) energy is conserved
( ) angular energy is conserved

Someone firing a rifle is pushed back by the rifle. This illustrates that
( ) momentum is conserved
( ) angular momentum is conserved
( ) energy is conserved
( ) angular energy is conserved

Internet Research Topics

See what you can find on the Internet about the following topics:

Manufacturing objects in space
Project skyhook
Spy satellites
Rail guns
Black holes
Ion propulsion
GPS
Lawn-chair Larry
Ion engine

Essay Questions

Automobiles and trucks are often designed with a tapered ("aerodynamic") rather than a blunt front surface. Why is this? What does the tapered front accomplish? How important is it?

Artificial-Earth satellites fly at different altitudes, depending on what they need to do. Describe the differences between low-Earth satellites, medium-Earth satellites, and geosynchronous satellites. For which applications would you use each?

Which would fall faster, a high diver doing a "swan dive" (in which he spreads out his arms and falls chest-first) or a diver with his arms pointed above his head going straight down? Why? Explain the relevant physics.

When you dive into the water, having your head down is much less painful than a "belly-flop." Discuss this in terms of the "area" that hits the water.

Science fiction movies and action movies often show things that don't fit in easily with the laws of physics. Give examples. Can you think of additional examples not described in this text book?

Black holes: Why can't we see them? How do we know they are there? Are any known to exist?